Improved Calculation of Core Loss with Nonsinusoidal Waveforms

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Abstract—**An extension to the Steinmetz equation is proposed, to enable estimation of hysteresis losses in magnetic core materials with nonsinusoidal flux waveforms. The new formulation is shown to avoid anomalies present in previous modified-Steinmetz-equation calculations of loss with nonsinusoidal waveforms. Comparison with experimental measurements in MnZn ferrite shows improved accuracy. The result may be optionally formulated in terms of an effective frequency and an effective amplitude, and options for defining these are discussed.**

I. Introduction

FOR the purpose of designing magnetic power devices, including electric machines, transformers, inductors, and other static reactors, loss in the magnetic material is often predicted using a power law equation [1], [2]

$$\overline{P_v(t)} = kf^\alpha \hat{B}^\beta \tag{1}$$

where $\hat{B}$ is the peak flux amplitude, $\overline{P_v(t)}$ is the time-average power loss per unit volume, and f is the frequency of sinusoidal excitation. A similar equation, but without the frequency dependence, was proposed by Steinmetz in 1892 [3], and so (1) is often referred to as the Steinmetz equation. Unfortunately, the Steinmetz equation, as well as the data provided by manufacturers of magnetic materials, is based only on sinusoidal excitation, whereas switching power converters and, increasingly, electric machines, can have very different waveforms. These nonsinusoidal waveforms result in different losses [4], [5]. DC bias can also significantly affect loss [6], [7], [8]. A better method of determining loss, accurate for a wider variety of waveforms, is needed. Our work is motivated primarily by applications to MnZn power ferrite materials. However, the results may be useful for other materials as well.

A standard method of analyzing core loss in more detail is to break it up into static hysteresis loss P_h, classical eddy current loss P_{cl}, and "excess loss" P_{exc} [9]

$$\overline{P_v(t)} = P_h + P_{cl} + P_{exc}. \tag{2}$$

The classical loss is a linear phenomenon, and depends on the square of the flux density. It can be directly calculated from geometry and bulk resistivity in ferrites, and, for power ferrites, it is typically small in the regions of interest. Static hysteresis loss is proportional to frequency, but typical values of β in (1) are around 1.5 to 2 in MnZn power ferrite materials [1]. This indicates that the bulk of the important losses in ferrite materials are, by definition, excess losses, those which are least well understood. Some important progress has been made in modeling excess losses in laminated alloys, based on considering eddy currents resulting from domain wall motion [10], [9]. The results for sinusoidal waveforms can be expressed as

$$P_{exc} = C(\hat{B}f)^\gamma \tag{3}$$

where C and γ are constants and $\gamma = 2$ or $\gamma = \frac{3}{2}$ under different assumptions. The model can also be used to predict loss with nonsinusoidal waveforms, with $\gamma = \frac{3}{2}$ giving good results for laminations and amorphous ribbon material [11], even though [11] improperly applies Fourier series to a nonlinear system.

Similar or equivalent approaches to modeling losses in ferrites, with $\gamma = 2$ (implicitly or explicitly) have sometimes been used [12], [13], [14]. It is also possible to use similar models that use linear dynamics and so have dynamic loss that depends on the square of the flux density, but that have more complex frequency dependence [15], [16], [17]. We have not used any of these approaches, for both practical and theoretical reasons. Firstly, in regions of high frequency and high flux density, where losses in power ferrites are primarily excess losses, the values of α and β in (1) are often different from either value proposed for γ, and α is smaller than β, by as much as a factor of two [1]. The complex frequency dependence in [16], [17] can help, but the results are still restricted to having the dynamic portion of the loss depend on $\hat{B}^2$, which may not agree with experimental data. This is not surprising, since the excess loss mechanisms in ferrite may be more directly related to hysteresis [18], [19] than is assumed in the models described in [10], [9]. It is not yet clear how to most accurately model the excess loss mechanism in ferrite. More detailed modeling of the dynamics of the hysteresis mechanism show promise to give the most accurate results [20].

However, all of the approaches using loss separation also have a practical disadvantage: they require extensive measurement and parameter extraction with a given material before they become useful. For practical design work, it would be preferable to be able to use manufacturer's data which is typically provided either in terms of coefficients for the Steinmetz equation, or in plots of loss for sinusoidal excitation, from which the coefficients of the Steinmetz equation can easily be extracted.

In [4], [5], [21], a modified Steinmetz equation (MSE) is developed to estimate loss with nonsinusoidal waveforms. The MSE appears to be what we are seeking, in that it requires no material characterization beyond the coefficients in the Steinmetz equation. It uses a combination of an effective frequency f_e and a repetition rate frequency f_r. The analysis is motivated by the idea that loss due to domain wall motion should depend on dB/dt. The MSE provides a good fit to experi-

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mental measurements of core loss with waveforms that deviate significantly from sinusoidal shapes in both ferrites and laminated alloys, without any additional parameter extraction [5], [21]. Thus, this is the previous work that has best achieved our objectives, and it is discussed in more detail in Section I-A.

In this paper, we refine the hypothesis of how loss depends on (dB/dt), in order to overcome anomalies in the behavior of the loss predicted by the MSE in [4], [5], [21]. The new hypothesis leads to a formula for calculating loss in the general case, which we term the generalized Steinmetz equation (GSE). The Steinmetz equation is shown to be a special case of the GSE. The GSE is also shown to avoid the anomalies of the MSE. A further benefit of the GSE is that it produces dc-bias sensitivity without the need to include additional measurements or curve-fitting functions for that purpose.

The GSE can be used directly, or can be used to generate an "equivalent frequency" and an "equivalent amplitude" that can be applied in the Steinmetz equation. These two parameters may be chosen in various ways if the only constraint is that the Steinmetz equation give the same result as the GSE. Thus, we need to introduce additional criteria. Appropriate criteria are discussed, and possible choices for effective frequency and effective amplitude are evaluated in Section IV.

A. The MSE

Based on physical understanding that loss depends on dB/dt, [5] averages dB/dt over a flux excursion to obtain

$$\left\langle \frac{dB}{dt} \right\rangle_B = \frac{1}{\Delta B}\int_0^T \left(\frac{dB}{dt}\right)^2 dt \tag{4}$$

where ΔB is the peak-to-peak flux amplitude and T is the period of the flux waveform. From this, an "equivalent frequency" is defined as

$$f_{eq} = \frac{2}{\Delta B^2 \pi^2}\int_0^T \left(\frac{dB}{dt}\right)^2 dt \tag{5}$$

The loss is then estimated with the modified Steinmetz equation (MSE):

$$\overline{P}_v = k f_{eq}^{\alpha-1}\hat{B}^{\beta} f_r \tag{6}$$

where $\hat{B}$ is the peak flux amplitude, $\overline{P}_v$ is the time-average power loss per unit volume and $f_r = 1/T$ is the repetition frequency [5].

One disadvantage of this formulation is that, although the dependence of loss on dB/dt is included, the manner in which it is included is not specifically matched to the frequency dependence of the Steinmetz equation. Thus, it is not clear from the derivation of the MSE that it will behave consistently with the simple Steinmetz equation (1). In fact, it does not. For example, consider a flux waveform consisting two sinusoidal components, at a frequency $\omega_0 = 2\pi f_0$ and at an integer multiple of that frequency $m\omega_0$

$$B(t) = A\left[(1-c)\sin\omega_0 t + c\sin m\omega_0 t\right]. \tag{7}$$

As the constant c is varied from zero to one, the core losses would be expected to gradually increase up to a factor m^{α} times the original losses. However, the MSE shows different behavior in this case.

The first surprise is that when we calculate f_{eq} (5) with c approaching one, the result is $m^2 f_0$, not the expected mf_0. However, despite this apparent overestimate of equivalent frequency, the power loss is typically underestimated: As c approaches one, the MSE predicts loss approaching $m^{2\alpha-2}$. Thus, the MSE underestimates by a factor of $m^{\alpha-2}$ the loss with a flux waveform comprising an m^{th} harmonic plus a negligible fundamental component. For a typical value of $\alpha = 1.5$, this error is a factor of $1/\sqrt{m}$, or 0.58 for a third harmonic. We see that if $\alpha = 2$, the error is zero. Thus, the MSE implicitly assumes loss proportional to f^2, while at the same time assuming loss proportional to f^{α}. Given this conflict, it is not surprising that some anomalies arise.

The region of c near one might seem to be only of academic interest, since many waveforms have harmonics that are small compared to the fundamental. However, the space of possible waveforms is very large, and it is impossible to explore them all, so it is worth paying attention to an anomaly that appears at one of the small subset of points that we have explored, and where we know what to expect. In addition, in multi-phase interleaved power converters with coupled magnetics (e.g., [22]), such waveforms can arise, and it is in this type of application, with waveforms that deviate dramatically from sinusoids, that a more general loss calculation method is most needed.

Beyond the problematic region where c is approaching one, at $c = 1$, there are at least two possible interpretations, because the period used for the integral in (5) and for f_r in (6) is not uniquely defined. Considering the waveform as a pure sinusoid at frequency mf_0 yields different results from considering it as a pure mth harmonic of frequency f_0. Neither choice is fully satisfactory. Using the latter interpretation results in a sudden jump in losses as c increases from $1-\epsilon$ to 1, whereas using the former interpretation results in different results from the Steinmetz equation for what is in fact a pure sinusoid, and it leaves open the question of whether any given pure sinusoid should be considered a harmonic of some lower frequency. Thus, in addition to the substantial underestimate of loss with small fundamentals as described above, the MSE has an undesirable dependence on what is chosen as the nominal fundamental period.

Another difficulty with the MSE is the treatment of waveforms with multiple peaks, such as those shown in Fig. 7, in which peak-to-peak amplitude is not adequately descriptive. As discussed in [4], it is possible to break up the waveform into multiple pieces, treating each separately. Although that approach appears to be effective, it would be more desirable to be able to directly calculate loss from the waveform.

II. The GSE: Hypothesis and Consistency with Steinmetz Equation

A fairly general hypothesis for instantaneous core loss is [23]

$$P_v(t) = P_d(\frac{dB}{dt}, B) \tag{8}$$

where P_d is an unknown power dissipation function. Assuming that P_d is a single-valued function of dB/dt and B may oversimplify the actual physical phenomena that, in general, depend on the time-history of the flux waveform as well as on its instantaneous value and derivative [9], [24]. However, if a function P_d can be found that matches experimental results well, calculating loss for arbitrary flux waveforms is then straightforward.

One possible form for P_d would be simply

$$P_v(t) = k_\gamma \left|\frac{dB}{dt}\right|^\gamma . \tag{9}$$

This dependence on dB/dt is physically plausible, as in the motivation for the MSE, but here there is some freedom to choose γ to attempt to make the results match the experimental exponents in the Steinmetz equation (1). However, the possible results can only have $\alpha = \beta = \gamma$, which contradicts the typical experimental result that $\beta > \alpha$, often $\beta \approx \alpha + 1$. If, however, we modify our hypothesis to

$$P_v(t) = k_1 \left|\frac{dB}{dt}\right|^a |B(t)|^b \tag{10}$$

we can show that this is consistent with the Steinmetz equation for sinusoidal waveforms if we choose $a = \alpha$ and $b = \beta - \alpha$.

$$P_v(t) = k_1 \left|\frac{dB}{dt}\right|^\alpha |B(t)|^{\beta-\alpha} \tag{11}$$

From this hypothesis, a formula that can be used to calculate loss for any waveform results directly:

$$\overline{P_v} = \frac{1}{T}\int_0^T k_1 \left|\frac{dB}{dt}\right|^\alpha |B(t)|^{\beta-\alpha} dt \tag{12}$$

We denote (12) as the generalized Steinmetz equation (GSE).

To check that the GSE is in fact consistent with the Steinmetz equation for sinusoidal waveforms, we substitute $B(t) = \hat{B}\sin\omega t$, resulting in

$$\overline{P}_v = k_1 \omega^\alpha \hat{B}^\beta \int_0^T \frac{1}{T} |cos\omega t|^\alpha |sin\omega t|^{\beta-\alpha} dt \tag{13}$$

With $T = 2\pi/\omega$, the integral here is independent of ω, and so this result can be made equal to the Steinmetz equation (1) with the appropriate choice of k_1

$$k_1 = \frac{k}{(2\pi)^{\alpha-1}\int_0^{2\pi} |cos\theta|^\alpha |sin\theta|^{\beta-\alpha} d\theta}. \tag{14}$$

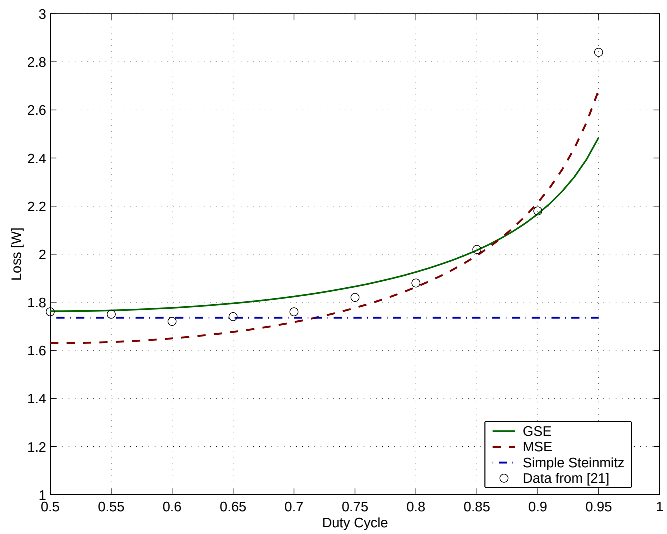

Fig. 1. Comparison of Loss predicted by the MSE, GSE, and Steinmetz equations with experimental data and Steinmetz coefficients from [21], for triangular waveforms with different duty cycles. The Steinmetz coefficients here are $\alpha = 1.3$, $\beta = 2.55$, and $k = 12$. The loss plotted is the total loss in a 17.3 cm^3 core. The fundamental frequency of excitation is 20 kHz, and the peak flux amplitude is 200 mT.

Thus, our hypothesis for the appropriate form of P_d in (8) has been shown to be consistent with the Steinmetz equation (1) for sinusoidal waveforms, and (12) may be considered a generalization of the Steinmetz equation for any waveform. It represents an improvement over the MSE in that different choices for the nominal period of the waveform do not affect the result, and, for sinusoidal waveforms, it can be expected to provide the same accuracy as the Steinmetz equation provides. An important limitation is that it is typically necessary to use different values of the parameters in the Steinmetz equation for different frequency ranges [1]. This shows that the GSE (12) will necessarily be limited in accuracy for a waveform containing harmonics at a wide range of frequencies.

III. Comparison with Experimental Measurements

A. Experimental data from the literature: triangular waveform with variable duty cycle

In [5], [21], the MSE's calculated loss is compared to experimentally measured loss for 3C85 ferrite with a variable-duty-cycle triangular flux waveform with a 20 kHz fundamental frequency. The MSE provides a good match to experimental results. These experimental measurements are compared with the predictions of the GSE, the MSE, and the Steinmetz equation in Fig. 1, using the parameters $\alpha = 1.3$, $\beta = 2.55$, and $k = 12$, for B in Tesla, f in Hz, and P_v in W/m^3, from [21]. From this plot it appears that both do reasonably well, with the GSE working better at duty cycles near 50% and the MSE working better at the most extreme duty cycle of 95%. This data is not sufficient to make any clear conclusions about the relative accuracy of the two methods.

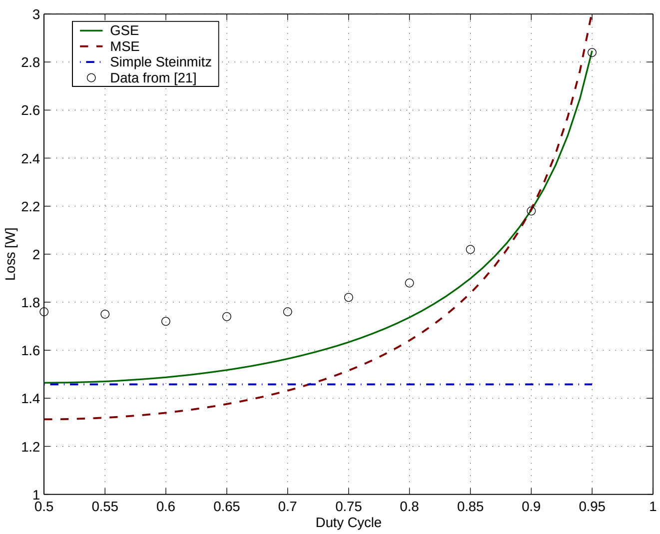

Fig. 2. Comparison of Loss predicted by the MSE, GSE, and Steinmetz equations with experimental data from [21] and parameters for the 100-200 kHz frequency range from [1] ($\alpha = 1.5$, $\beta = 2.6$, and $k = 1.5$. The loss plotted is the total loss in a 17.3 cm^3 core. The fundamental frequency of excitation is 20 kHz, and the peak flux amplitude is 200 mT.

The underestimate provided by either method for the extreme point at 95% duty cycle could be explained by the fact that the Steinmetz parameters used only work well over a limited frequency range; [1] recommends different parameters for frequencies in the 100 kHz to 200 kHz range, where, in particular, $\alpha = 1.5$ gives a better fit. The steep slope in a 95% duty-cycle, 20 kHz excitation is the same as the slope 50% duty cycle operation at 200 kHz, so these parameters are arguably more appropriate. In Fig. 2, the same experimental data is compared to the predictions of the three models with the 100 kHz to 200 kHz parameters provided by [1]. The fit near 50% duty cycle is poor, for either model, but the fit at high duty cycle is improved, particularly for the GSE, which now fits the last two data points very well. This shows that both, and particularly the GSE, can do well, but are limited by the accuracy of the underlying Steinmetz model for sinusoidal loss. For waveforms that have harmonic energy over a wide range of frequencies, this limitation is most severe, since, for any given waveform, a single set of parameters must be used, and the strategy of using different parameters for different frequency ranges as in [1] is not as straightforward to apply, even though the results in Fig. 1 and 2 show that it can work well.

B. New experimental measurements: sum of harmonically-related sinusoids

The theoretical discussion in Section I-A of the loss with a flux waveform consisting of two harmonically related sinusoids indicated that the GSE had some advantages over the MSE in this situation, but this needed to be verified experimentally. To obtain the necessary experimental data, we performed core loss measurements using two windings of six turns each on a toroidal core of 3C85 MnZn power ferrite

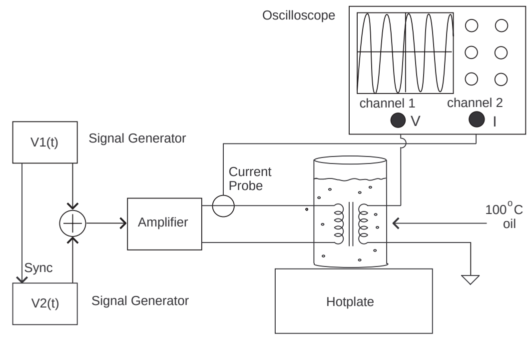

Fig. 3. Loss measurement experiment.

(Philips), as shown in Fig. 3. The outputs of two synchronized function generators are combined and fed to an amplifier (Hafler P4000, designed for audio applications but with a power bandwidth of 200 kHz). Current is sensed with a 50 MHz ac current probe (Tektronics P6021), and a 60 MHz bandwidth 200 MSa/s oscilloscope (Agilent 54621D) is used to acquire, multiply, and average voltage and current waveforms, using averaging to obtain 11-bit resolution.

The core is submerged in a 100°C vegetable oil bath in order to perform measurements at a constant temperature consistent with the primary data given in [1] and with the measurements in [5], [21]. The amplifier is turned on only briefly when each data point is measured, in order to be sure the core remains at thermal equilibrium with the oil bath. The two-six turn windings used Teflon-insulated stranded wire (AWG 30 constructed from 7 strands of AWG 38) to minimize eddy current loss and capacitive effects that might affect the measurement. Each winding was distributed uniformly around the core. Initial experiments showed that submerging the core in room-temperature oil affected the measurement by about 5%, indicating a 5% error. Blocking the hole in the center of the core with a rubber plug eliminated this effect, or reduced it to an unmeasurable level. Apparently, significant displacement currents were being induced in the oil, circulating in a closed path through the center hole, and they were blocked by plugging the hole. The core has an inner diameter of 19.5 mm, an outer diameter of 39 mm, and a thickness of 12.5 mm.

Fig. 4 compares the predictions of the MSE and the GSE for a flux waveform

$$B(t) = A\left[(1-c)\sin\omega_0 t + c\sin(3\omega_0 t + \phi)\right]. \qquad (15)$$

This is the same waveform as (7) with $m = 3$ selected (third harmonic), and the possibility of a phase shift ϕ added. Fig. 4 shows that as the fraction of third harmonic, c, rises from zero to about 0.15, the curves are very similar for both models. However, depending on phase ϕ, they begin to deviate above that point, with the most dramatic separation occurring above $c = 0.6$. An interesting difference is that above $c = 0.6$, the GSE predicts almost no sensitivity to phase, whereas the MSE retains that sensitivity through most of the range of values of c.

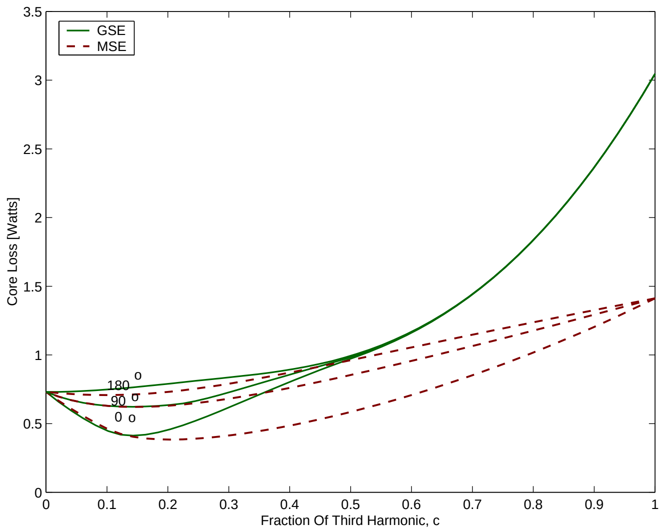

Fig. 4. Loss predicted by the MSE and the GSE for a flux waveform composed of two harmonically related sinusoids (15). The example here is based on loss in the 3C85 MnZn ferrite core used in the experiment shown in Fig. 3 with 20 kHz and 60 kHz sinusoids, with the fraction of 60 kHz described by the parameter c of (15). The three curves for each model are for different phase relationships between the sinusoids, as marked on the plot and defined by ϕ in (15).

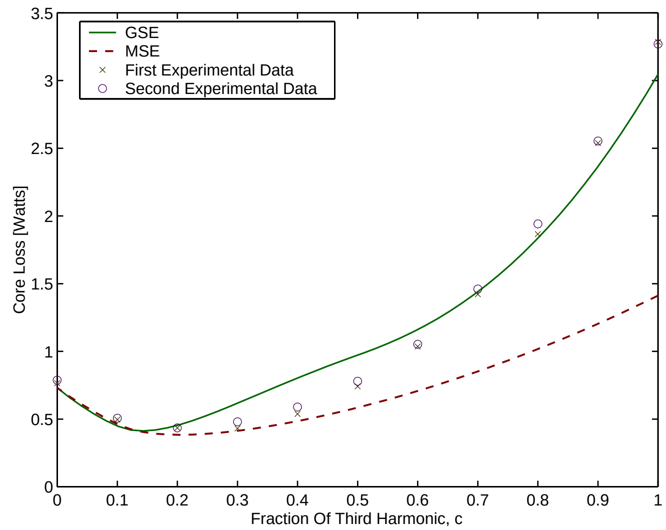

Fig. 5. Comparison of loss predicted by the MSE and the GSE to experimental data for a flux waveform composed of two harmonically related sinusoids (15), with phase $\phi = 0$. The two sinusoids are at 20 kHz and 60 kHz, with a maximum amplitude of 200 mT.

We performed two sets of experiments, designed to examine experimentally the differences between the GSE and MSE and determine which is more accurate. In the first experiment, we held phase fixed and scanned through a range of relative amplitudes of the two sinusoids c. Because the zero-phase curve showed the most interesting theoretical behavior, dipping substantially before rising, and with significant differences between the GSE and MSE arising relatively early, we chose to perform the measurements with phase $\phi = 0$. The results of this first experiment are shown in Fig. 5. As expected, the MSE and the GSE both fit well for the pure fundamental ($c = 0$), and at $c = 1$ the MSE fits poorly, but the GSE fits well. The experimental data is about 5% high at $c = 0$ compared to either model, and also 5% above the GSE at the other endpoint. This could be attributed to batch-to-batch variation in the ferrite relative to the batch(es) measured to produce the Steinmetz parameters in [1], inaccuracies in using databook values for effective volume and area, or to a systematic error in our measurements. However, the consistency of the 5% error between the endpoints confirmed that the value of α that we used, 1.3, accurately described the loss behavior for pure sinusoids.

Throughout the range of $c > 0.6$, the GSE fits much better than the MSE, which has an error of about 57% at the $c = 1$ endpoint. However in the range of $0.3 \leq c \leq 0.5$, the GSE has significant error (up to 40%), while the MSE is more accurate. The maximum error of the GSE is smaller than the error of the MSE, and it is the better model over a wider range of values of c, but the middle range of harmonic amplitudes, where the MSE is more accurate, may be important in a greater number of applications.

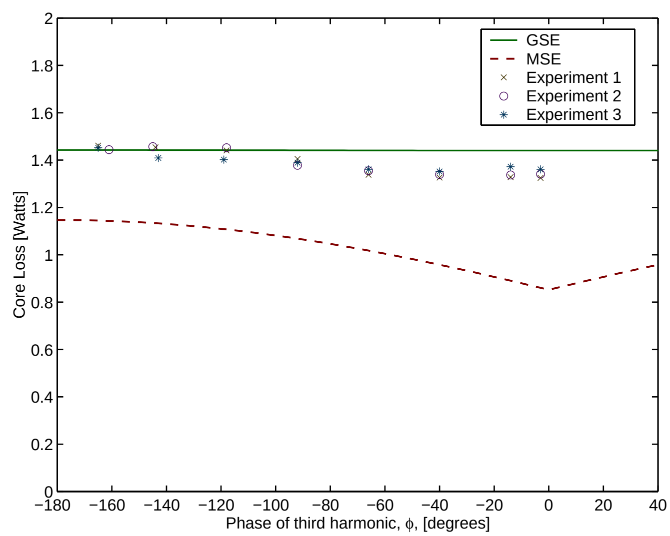

Fig. 6. Comparison of loss predicted by the MSE and the GSE to experimental data for a flux waveform composed of two harmonically related sinusoids (15), with fixed amplitudes of a ratio $c = 0.7$ and with phase ϕ varied.

For the second set of experiments, we held the amplitudes fixed and varied only phase. Looking at Fig. 4, we chose to do this at $c = 0.7$ because the two models are dramatically different at that point, with the GSE predicting no variation with phase, and the MSE predicting significant variation. Fig. 6 shows the results of that experiment. Although the loss does show some sensitivity to phase, the variation is only about 8%, whereas the MSE predicts a 30% variation. The GSE prediction is more accurate both in predicting little variation with phase, and in predicting the magnitude of loss.

C. Discussion

Overall the GSE is more accurate than the MSE in most situations, and is useful over a broader range of conditions. In situations with a small-amplitude fundamental-frequency component in the flux waveform, such as might be encountered in some integrated magnetic components, the MSE fails to provide useful predictions, and the GSE is clearly superior.

However, the GSE does have significant limitations. Firstly, it depends on having set of Steinmetz parameters valid for the full range of frequencies of interest. As shown in Figs. 1 and 2, the use of different parameters may be necessary to cover a wide range of frequencies in the flux waveform spectrum. While an examination of the predominant frequencies involved in producing loss can probably adequately guide the selection of parameters for that frequency range, a less ad-hoc approach would be desirable.

Also, we have found one range of harmonic amplitudes and phases where the GSE can have as much as 40% error in predicting loss: the range where the third harmonic flux amplitude is near to, but smaller than the fundamental flux amplitude (around $0.3 \leq c \leq 0.5$), with phase equal to zero. Because the GSE relies only on the Steinmetz parameters, and does not attempt to model hysteresis processes in a physically meaningful way, it is not surprising that it does have some limitations. The deviation between the GSE and the MSE begins around $c = 0.15$, which is also about where the BH trajectory begins to have minor loops, because the flux waveform has multiple peaks. This may indicate that hysteresis modeling explicitly taking these minor loops into account would be necessary to overcome this limitation of the GSE.

Despite its limitations, we believe that the GSE is in most cases the most useful tool available to magnetics designers who need to predict losses in MnZn ferrites with nonsinusoidal flux waveforms. It gives accurate predictions for a wide range of conditions, and provides a simple way to make these predictions requiring no material characterization beyond the Steinmetz parameters, which are often available from core manufacturers.

Future experimental work should expand the range of test conditions, including more complete data with different parameter values for excitation by two harmonically-related sinusoids, testing of dc-bias effects, and testing of different core materials.

IV. Effective Frequency and Effective Amplitude

We define effective amplitude as an amplitude that may be substituted into a simple formula, originally intended for simple waveforms, to enable calculation of loss arising from arbitrary waveforms. A familiar example of an effective amplitude is rms amplitude, which, for current in a resistor, allows the use of the dc loss formula $I_{dc}^2 R$ to calculate loss for any waveform, if the effective amplitude of current, I_{rms}, is substituted for I_{dc}. The use of rms amplitude for this case is dependent on the specific loss mechanism in the resistor—that of Joule heating. Different loss mechanisms lead to different formulations for the appropriate type of effective amplitude. For example, in a diode, where voltage drop is not proportional to current, rms current is not appropriate for calculating loss. If the voltage drop is approximately constant, average current is the appropriate effective amplitude.

Effective frequencies are less commonly used than effective amplitudes, but can be useful for frequency-dependent winding losses [25], [26], [27]. Again, the calculation of effective frequency is specific to the loss mechanism, in this case eddy-current losses in conductors that are small compared to a skin depth.

For use in the Steinmetz equation, what is needed is an effective frequency f_e and an effective amplitude $\hat{B}_e$, defined such that

$$\overline{P}_v = \frac{1}{T}\int_0^T k_1 \left|\frac{dB}{dt}\right|^\alpha |B(t)|^{\beta-\alpha} dt = k f_e^\alpha \hat{B}_e^\beta. \qquad (16)$$

This is one equation with two unknowns, so we have the freedom to define either one arbitrarily, and then simply solve (16) to define the other. However, the result should ideally make some sense in that the effective frequency should relate to the rate at which things change in the waveform, and the effective amplitude should relate to the amplitude of the waveform features.

One possibility would be to use one of a number of standard, known measures for amplitude to compute $\hat{B}_e$, such as rms, peak-to-peak amplitude, or average absolute value of amplitude. However, this approach can lead to counter-intuitive definitions of effective frequency.

If we use peak-to-peak amplitude, the difference in loss induced by the two different waveforms shown in Fig. 7 would have to be accounted for by differences in effective frequency, since the peak-to-peak amplitude is the same. Yet these waveforms intuitively should have similar if not identical effective frequencies.

Next we consider the other conventional amplitude measures, average absolute value and rms, which are special cases of the general expression

$$\hat{B}_e = \left(\frac{1}{T}\int_0^T |B(t)|^b dt\right)^{\frac{1}{b}} \qquad (17)$$

where $b = 1$ for average absolute value and $b = 2$ for rms. The results will correlate well with an intuitive idea of amplitude for any reasonable value of b, but, specifically for the Steinmetz equation, the calculations are greatly simplified and more appealing with $b = \beta$. Then, solving (16) for f_e results in

$$f_e = k_2 \left[\frac{\int_0^T \left|\frac{dB}{dt}\right|^\alpha |B(t)|^{\beta-\alpha} dt}{\int_0^T |B(t)|^\beta dt}\right]. \qquad (18)$$

This is a reasonable, defensible choice. However, it arguably still gives results that do not properly correlate $\hat{B}_e$ with amplitude-related changes and f_e with rate changes. Consider, for example, the waveforms in Fig. 8. The peak-to-peak

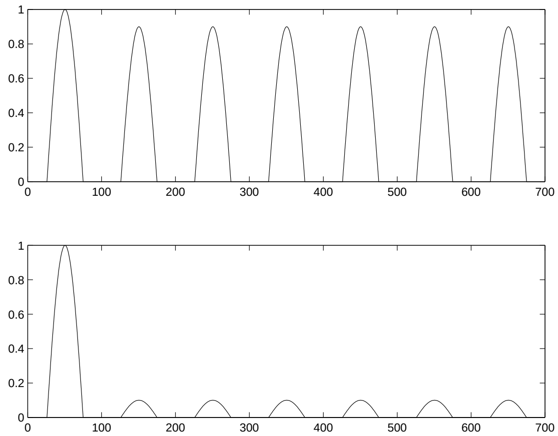

Fig. 7. Two waveforms with the same peak-to-peak amplitude, but that induce different loss, and intuitively should have the same effective frequency.

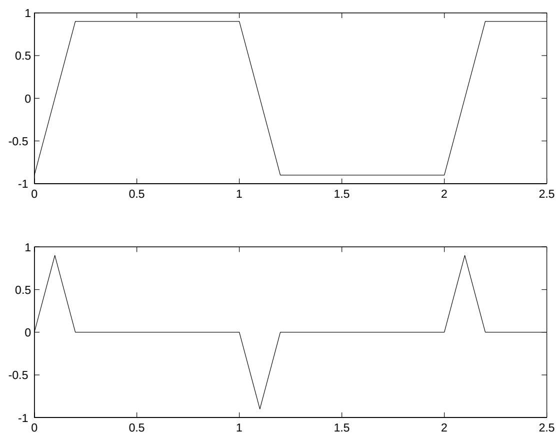

Fig. 8. Two waveforms that are expected to induce the same loss, and that intuitively should have the same f_e and the same $\hat{B}_e$ (the same effective frequency and effective amplitude).

amplitude is the same, and the waveforms change the same amount at the same rate. It is merely the sequence that is different. Calculation of loss using (12) gives the same result for either waveform, and it would make sense for the effective amplitude and the effective frequency to also be constant between the two. But the formulae (17) and (18) give drastically different results for each. Changing the value of b in (17) does not help.

Another approach would be to use an established measure for effective frequency. In fact, the "equivalent frequency" proposed in [5] is related to a well established measure of effective frequency used in winding-loss calculations [25], [26], [27]. A generalized form of the effective frequency used in [25], [26], [27] would be

$$f_e = \frac{\left(\frac{1}{T}\int_0^T \left|\frac{dB}{dt}\right|^a dt\right)^{\frac{1}{a}}}{2\pi\left(\frac{1}{T}\int_0^T |B(t)|^b dt\right)^{\frac{1}{b}}} \tag{19}$$

Alternatively, some other measure of magnitude for B could be used in the denominator of (19), as is done in the "equivalent frequency" used for the MSE (5), where peak-to-peak amplitude is used and $a = 2$ [5]. In addition, (5) differs by omitting the division by T, and omitting the a^{th} root. In the case of a single sinusoid, these two differences cancel each other, and the result is an "equivalent frequency" equal to the actual frequency, but, as discussed in Section I-A, for a waveform consisting primarily of a harmonic, the "equivalent frequency" ends up counterintuitively mismatching the actual frequency of the main component of the waveform. The result is still useful in the MSE, but it is not the effective frequency we wish to use here.

The most natural way to use (19) with (12) would be to use $a = b = \alpha$ in this formulation. However, the net result is a more complicated overall calculation of $\hat{B}_e$ and f_e than (17) and (18) and the result is subject to all the limitations inherent in whatever measure of amplitude of B is used in the denominator of (19).

Given the limitations of the approaches discussed above, a better measure of effective frequency is desired. The following criteria are necessary (though perhaps not sufficient) in order for a set of effective frequency and effective amplitude measures to work for predicting loss and to overcome the objections to other approaches discussed above:

- The amplitude and frequency of any pure sinusoid should match the conventional measures (or at least be proportional to them).
- The loss calculated according to the Steinmetz equation should match the loss calculated according to the GSE ((8) should be satisfied).
- For waveforms such as those shown in Fig. 7, the difference should be primarily in effective amplitude rather than effective frequency.
- For waveforms such as those shown in Fig. 8, both the effective amplitude and the effective frequency should both match between the two cases.
- An addition of dc bias may reduce effective frequency, or may increase effective amplitude, but it should not increase effective frequency or decrease effective amplitude.

We propose two approaches that each satisfy most of these criteria:

A. *Effective parameters: first option*

An effective amplitude can be defined as one-quarter of the total integral of the flux change. For waveforms that do not have multiple peaks, this corresponds to peak-to-peak amplitude.

$$\hat{B}_e = \frac{1}{4}\int_0^T \left|\frac{dB}{dt}\right| dt \tag{20}$$

Then effective frequency is defined by (16).

This approach satisfies all the criteria except for dc bias. The effective amplitude is not affected by bias, which does not directly pose a problem, but unfortunately it does means that effective frequency must increase with dc bias in order to account for the increased losses observed in practice and predicted by (12).

B. Effective parameters: second option

Alternatively, an effective frequency can be defined as follows:

$$f_e = \frac{\int_0^T \left|\frac{d^2B}{dt^2}\right| dt}{2\pi \int_0^T \left|\frac{dB}{dt}\right| dt} \tag{21}$$

Then effective amplitude is defined by (16).

The results of this approach satisfy all the criteria described above, and may be used as a definition of effective frequencies and amplitudes that allow the Steinmetz equation to deliver results identical to those of the GSE (12).

V. Conclusion

The generalized Steinmetz equation (GSE) (12) is a new formulation that is fully consistent with the Steinmetz equation for sinusoidal waveforms, but allows the calculation of loss for any waveform. Experiments show that it is often more accurate than the MSE method, particularly for waveforms with small fundamental-frequency amplitudes, where experimental results show the MSE can err by 57% where the GSE gives error of only 5%. Although the accuracy of the GSE is typically within 5%, errors as high as 40% can arise in some situations. Nonetheless, we believe the GSE to be the most practical method for calculating core loss with non-sinusoidal waveforms in most high-frequency magnetics design work, because it requires only readily available sinusoidal loss data, in the form of parameters for the Steinmetz equation characterizing the material.

Although the GSE (12) is the only result needed to calculate loss for nonsinusoidal waveforms, it may also be elucidative to describe a waveform in terms of an effective frequency and an effective amplitude. Many standard approaches to defining effective frequency and effective amplitude have been shown to be inappropriate for describing loss in magnetic materials. Two new formulations for effective frequency and effective amplitude are proposed, one of which (21) is found to meet all the criteria we propose for a good way to define those parameters in this application.

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