Optimal Core Dimensional Ratios for Minimizing Winding Loss in High-Frequency Gapped-Inductor Windings

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***Abstract*— Numerical techniques are used to find the inductor core dimensional ratios that minimize winding loss. It is shown that common core shapes result in significant excess losses, even if the shape of the wire winding is optimized. A design example demonstrates the practical implications of this technique for choosing cores—a standard core with dimensional ratios close to optimum provides a 32% savings in power loss compared to another popular core shape. Further improvements in power loss could be achieved by using optimized core shapes.**

Improvements to software for shape-optimization of windings are described, including accounting for different turn lengths at different radii, the ability to select gaps in different core legs, and better approximations of three-dimensional field geometry.

I. Introduction

THE fringing field from the air gap in an inductor can cause severe eddy-current losses in the winding [1], [2], [3]. Due to the high intensity of the fringing field near the inductor gap, and the dependence of loss on the square of the magnetic field, these losses can be greatly decreased by placing the wire windings away from the centerpost gap [2], [4]. Although the effect of fringing fields on gap reluctance is most severe for large gaps, the effect on winding losses is almost independent of gap length; it must considered whenever the reluctance of the gap is significant compared to the reluctance of the core [5]. In [4] a numerical method was presented to optimize the shape of a wire winding, taking into account the two-dimensional (2-D) shape of the field, the effect of the winding shape on the field shape, and the effect of the winding shape of both resistive and eddy-current losses. Inductors designed using an optimized winding shape have been shown to achieve lower losses than those in a distributed gap inductor [4], [6], [7].

In [4], [6], the optimizations of wire placement were conducted for a square winding window, where the aspect ratio $b_w/h_w = 1$. However, few inductor cores are made with a square winding window. The window breadth, b_w, is typically 1.5 to 5 times larger than the window height h_w. This paper investigates the effect of aspect ratio on optimized-shape inductor windings. For a variety of different cases, we can find an optimum aspect ratio that will give minimum winding losses with a shape-optimized winding. This optimum aspect ratio is a function of frequency and other parameters. Significant increases in power loss result if one deviates from this optimal aspect ratio.

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We have also overcome other limitations of the shape optimizations in [4], [6], [7]. The curvature of the winding was neglected in these optimizations. Although it has been shown that these curvature effects may typically be neglected for calculation of the field [8], a more basic effect is still important for shape optimization: Turns placed at a large radius from the centerpost are longer and thus have larger resistive and eddy-current losses. In some geometries, such as planar cores with long, narrow, straight, tunnel-like winding windows, the original analysis is valid. However, for most inductors an accurate optimization of wire placement must consider the effect of radius on wire length and loss. Optimization software taking this effect into account has been developed. Other new features include the option to select gaps in all legs of an E-core, or only in the center leg; restricting wire placement to the region defined by the bobbin rather than making use of the whole core window; and a more accurate approximation of 3-D fields by using several 2-D simulations according to the method developed in [9].

In Section II, we study the effect of aspect ratio for the simple analysis neglecting curvature. In Section III, we re-examine the effect of dimensional ratios with the effect of radius taken into account. Section IV presents the improvements that have been made to the shape optimization software. In Section V, we show that choosing a standard core considering dimensional ratios leads to substantially improved performance in an example design.

II. Optimal Aspect Ratio

To study the effect of aspect ratio, the COOS software is used [10], [4], [6]. We assume that fine litz wire is used (smaller than a skin depth), so that eddy-current losses can be approximated by

$$P_{pe} = \frac{\pi \cdot \omega \cdot |\hat{B}|^2 \cdot \ell \cdot d^4}{128 \cdot \rho_c} \quad (1)$$

where ρ_c is the resistivity of the conductor, d is the diameter of the wire, ℓ is the length of the conductor, $\hat{B}$ is the peak value of the ac field perpendicular to the axis of the wire at a frequency ω [11], [4]. Resistive losses, $P_r = I^2_{rms}R_{dc}$ can be found from a simple dc resistance calculation. The optimization uses a fixed, specified litz-wire strand diameter, and adjusts the number of strands in a bundle, as well as the positioning of the wire in the window, until it finds the arrangement that give minimum total loss. Using fewer strands leads to lower eddy-current loss, both because lower window utilization allows doing better as far as keeping wire out of high-field regions, and because it simply

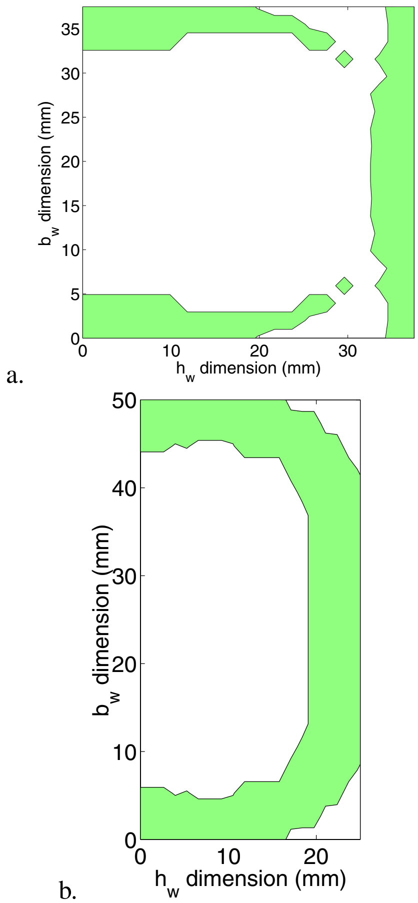

Fig. 1. Comparison of two shape-optimized winding designs with different window aspect ratios. Both are for a 50 kHz 0.5 A rms current in a 100-turn litz-wire winding using 36 AWG (0.127 mmm) strands. a. (top) Winding loss = 133 mW. b. (bottom) Winding loss = 109 mW. Wire is placed in the shaded area. The gap is in the middle of the left side.

means a smaller amount of copper subjected to loss-inducing high-frequency fields. On the other hand, fewer strands leads to higher dc resistance. The optimization balances this tradeoff.

Fig. 1 demonstrates the effect that the aspect ratio has on the optimized winding shape. The same inductor is optimized (for the same frequency) in the two figures, except the aspect ratio b_w/h_w is varied. The perimeter of the winding window is also kept constant to keep the volume of core magnetic material constant. It is clear that the 1:1 ratio in the first figure is not optimal, because the second aspect ratio leads to lower losses. This result is specific to this particular inductor; however, general conclusions can be found by optimizing a variety of inductors and varying the frequency and aspect ratio.

To find more general results for the optimal aspect ratios and put the information in a form useful to designers, we ran 3200 inductor winding shape optimizations over a range of frequencies and aspect ratios. To describe the results independent of specific design details, Fig. 2 plots the aspect ratio as a function of a_n, where a_n is the area used by the winding normalized to a filled square winding window with the same perimeter. The best aspect

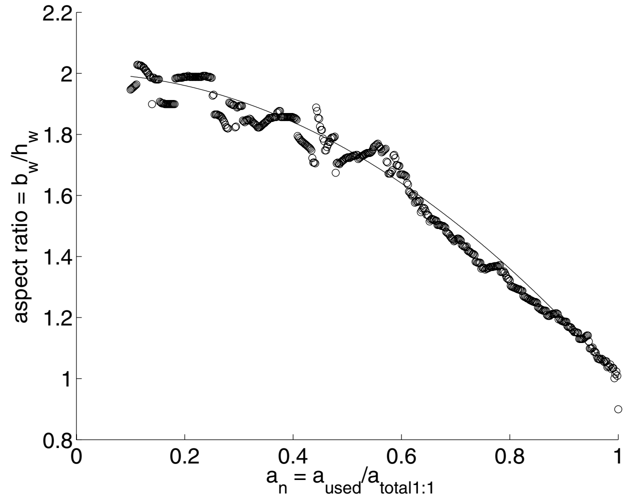

Fig. 2. Optimum winding window aspect ratios based on the simple 2-D analysis. The numercial results (circles) are shown with a curve fit (2) to these results.

ratio found for a given a_n is marked as a circle. In general, the window area used in an optimum-shape design, a_n, decreases with increasing frequency. We see that in the low frequency limit, the optimum winding window shape is a square. At high frequencies, the optimum aspect ratio approaches $b_w/h_w = 2$. A curve fit reveals a very simple expression for the optimum aspect ratio:

$$\frac{b_w}{h_w}|_{opt} = -a_n^2 + 2 \tag{2}$$

For small values of a_n, this function is a worse fit to the data, and the data is not monotonic. This can be explained by the resolution used—the winding window is divided up into a 20×20 grid for the determination of optimal shape. At low a_n (higher frequency), the area of the winding window used is small, and thus the discretization error from the 20×20 matrix is more apparent.

The optimum aspect ratio has been found; however, one would like to know how important it is to be near the optimum. How much does one pay (in power loss) for a design that is not optimal? Fig. 3 shows a contour plot of the percentage of power loss greater than at the minimum. It can be seen that the aspect ratio is, in fact, significant. For example, if an inductor with a square winding window is used at high frequencies the power loss will be 75% worse than if that same inductor had been optimized with a winding window of aspect ratio two, even with the winding shape individually optimized for each.

III. Accounting for Centerpost Radius

A more accurate optimization can be obtained by taking into account the radius of the inductor centerpost. Now, we no longer assume that h_w is small compared to that radius. Instead, we factor in the length of each turn as determined by its position in the winding window. A turn near the outside of the window is longer than one that is wound near the centerpost. When the shape is optimized according to the original algorithm, the average

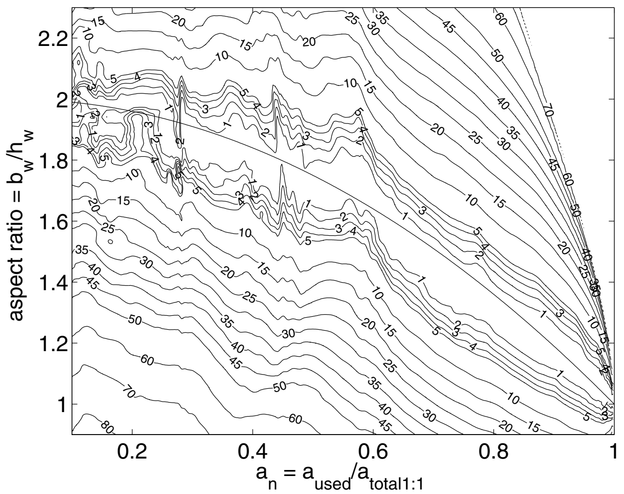

Fig. 3. Optimum winding window aspect ratios based on the simple 2-D analysis, as in Fig. 2, but with contour lines added to show the percentage of winding loss increase from the optimum.

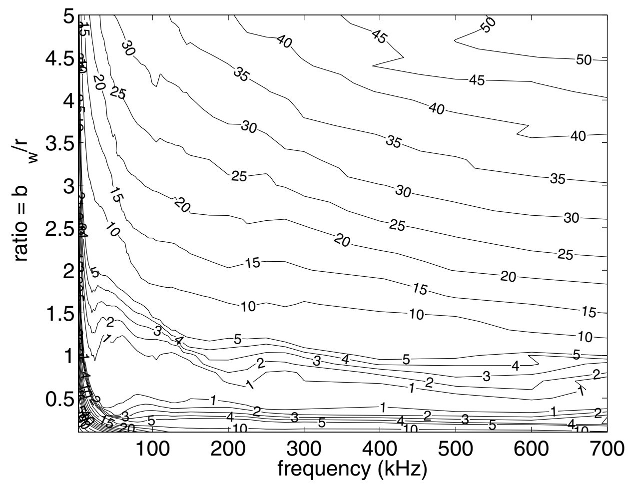

Fig. 4. Optimum ratio of window breadth to centerpost radius, showing percentage winding loss increase from the optimum. The vertical scale is applicable to any design, whereas the frequency scale would shift for other example. The example ploted here used strand diameter $d = 0.1$ mm, packing factor F_p = 0.85, and radius r = 10 mm.

length of a turn increases because wire is placed away from the gap to reduce eddy-current losses. Resistive losses scale with the total length of the wire. The eddy current losses are calculated by

$$P_e = \frac{\pi \cdot \omega^2 \cdot F_p}{128 \cdot \rho_c} \cdot \sum_{i=1}^{N} |B_i|^2 \cdot 2\pi r_i \tag{3}$$

where F_p is the packing factor, N is the number of turns, and r is the radius of a turn of wire.

With an optimization program based on this simple modification, we could do the same optimizations as above to more accurately find the optimal aspect ratio. However, we find that this model shows us that a detailed study of this aspect ratio is not necessary. This is because making the window height h_w very large does not decrease power loss, as placing wire far from the centerpost would lead to large resistive losses. We need only make the window height large enough so that the winding has as much space as it needs to achieve low power loss — increasing h_w beyond this point does not decrease power loss, and is in fact actually undesirable because the size of the core will have to be increased, leading to more magnetic material which would increase both cost and core losses.

However, choosing b_w is more complicated. We find that the choice of b_w which leads to the lowest power loss is a function of the centerpost radius. A different dimensional ratio, b_w/r is now the parameter of interest. We can apply the same technique as in the last section. A figure of merit for this analysis is $M = b_w \cdot P_{winding}$, where we normalize to the window width b_w. Minimizing M gives the optimal dimensional ratio at a given frequency. For this analysis, we set h_w large enough that the optimized shape does not use the full height available.

The results for the optimum b_w/r are shown in Fig. 4. We see that the 5% band is very wide, meaning that a wide range of ratios will work well. A ratio of $b_w/r = 0.5$ is close to optimum for the full frequency range, but this is not critical—a ratio of one ($b_w = r$) is within the 5% band. However, most standard cores made today use a much larger ratio—closer to 4 or 5. As one can see from Fig. 4, this can result in power loss 40% higher than it could be, even if the wire winding shape is optimized.

IV. Additional Software Improvements

An improved version of the COOS software [4], called shapeopt has been developed. The new version has been modified to take into account the use of a bobbin, which has been an important step in making this simulation tool accurately predict the results for a real design. In addition, we have expanded the utility of the program by allowing the designer to optimize for several different gap locations. This addition was made possible by incorporating the extended reluctance path model as discussed in [9]. Now, the user can specify whether they will be placing the gap in the centerpost, the outer legs, or both the centerpost and the outer legs. Our initial investigations with this tool have indicated that inductors with gaps in both the centerpost and the outer legs have the potential for lower winding loss than with other gap placements. However, more simulations will be necessary to explore the possibilities for lower loss in inductors with these gap locations.

In addition to improving the modeling accuracy and utility of the software, we have also made this simulation tool easier to use. The addition of a graphical user interface (Fig. 5) greatly simplifies the design process. The interface allows the inductor designer to easily enter the geometry and design constraints for the inductor into text boxes. The user can then press the Run button and shapeopt generates the optimized shape and power loss predictions. This software can now be downloaded from our website [10]. A web-based version, similar to the web-based litz-wire optimization program described in [12], is under development for those users who do not have access to MATLAB.

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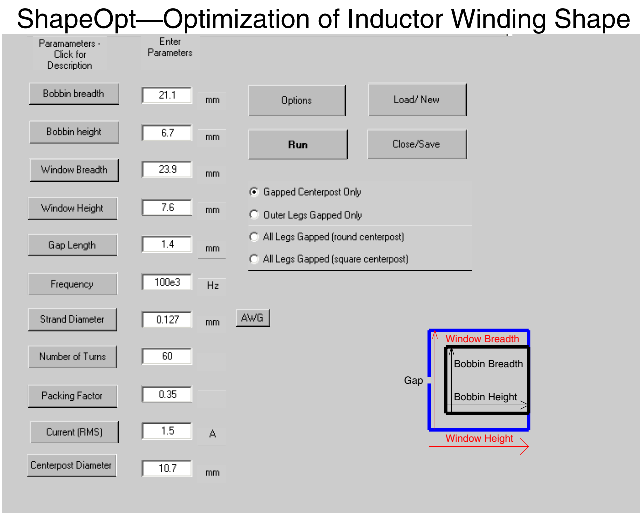

Fig. 5. Graphical user interface of the shape optimization program.

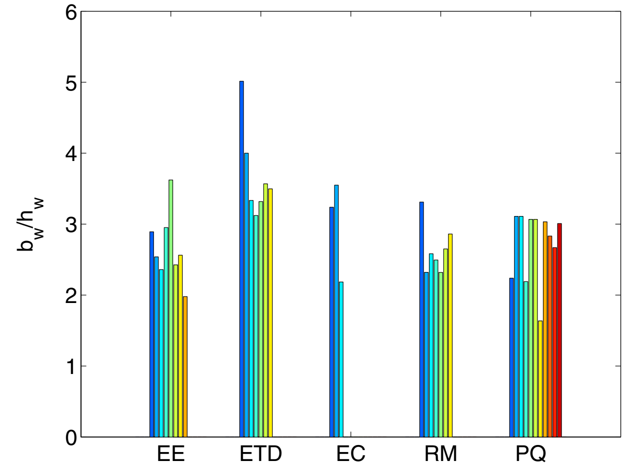

Fig. 6. Typical core aspect ratios b_w/h_w. Optimal values range from one to two, as shown in Figs. 2 and 3.

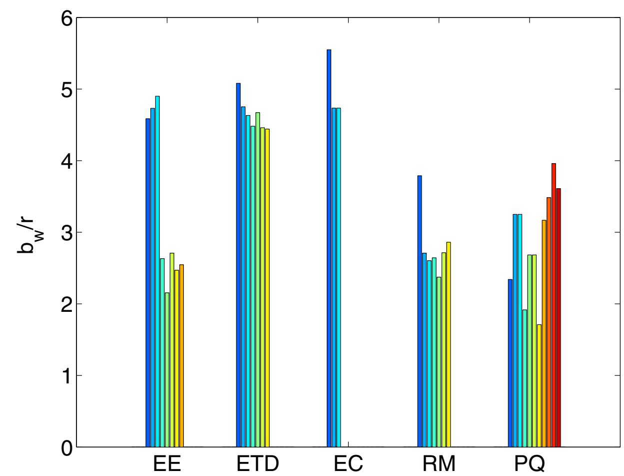

Fig. 7. Typical core aspect ratios b_w/r. A value of 0.5 is optimal for most designs. As shown in Fig. 4, values around one are almost as good, but losses increase with higher aspect ratios.

These software improvements have been used to design several example inductors which are presented in Section V. We found that it can be extremely important to account for the bobbin which is used in winding a real inductor. Our original simulation assumed that the entire winding window would be available for the placement of wire, including placing the wire directly adjacent to the core gap. In practice, the use of a bobbin reduces the available area and prevents wire from being placed in the high B field region near the gap. For non-optimized windings filling the bobbin, this offset is enough to greatly reduce the ac resistance compared to the ac resistance that would result if the wire were placed immediately adjacent to the gap. For example, for the full-bobbin design discussed in Section V, filling the window with wire instead of filling the bobbin would approximately double the winding loss. For shape-optimized designs, however, this modification does not make a significant difference, because no wire is placed immediately adjacent to the gap in either case.

In addition, we modified shapeopt to take into account flux paths that travel out of the plane of the inductor core, as discussed in [9]. This improvement allows us to more accurately model a three-dimensional inductor with a two-dimensional simulation. This new model was used for our example designs, however, we found that this modification was not critical for predicting loss for these specific inductors. However, the extended reluctance model can make a very large difference for inductors that have gaps in the outer core legs [9]. Further investigation will be necessary to explore the potential for shape optimized wire windings for this class of inductors. However, initial findings have indicated that substantial reductions in ac resistance are possible.

V. Example Design

A look at a core catalog reveals that most cores have dimensional ratios that are far from optimal, as can be seen in Fig. 6 and 7. To test our findings about optimal aspect ratios, two inductors with a similar product of winding area and core area ($W_a A_c$) were optimized (including winding shape) for the same application. As can be seen in Figs. 6 and 7, RM cores are consistently closer to the optimum dimensional ratios than ETD cores. For an application in which an inductor has a current of 1.5 A_{RMS} and a voltage of 400 V_{RMS}, inductors based on RM12 and ETD34 cores were each optimized based on gapping only the centerpost. This practical case does not fall purely into one of the categories analyzed in Sections II or III. The turn length varies significantly with position, so the analysis of Section III applies, and the enhanced optimization taking this effect into account is used. However, the window aspect ratio b_w/h_w also matters in these examples, as the optimal winding shape does use some turns at the maximum radius position. Thus, we can expect that favorable values of both dimensional ratios will be important; the RM core has better values of both.

As shown in Table V, the calculated performance of the RM

TABLE I

PERFORMANCE COMPARISON INDUCTORS USING FOR SIMILAR-SIZE CORES WITH DIFFERENT ASPECT RATIOS.

All designs are for a 424 μH inductor with a 1.5 A 100 kHz sinusoidal current. All use 36 AWG strand size litz wire, but the number of turns is optimized to minimize total loss, including all winding losses and core loss.

Design	WaAc	b_w/h_w	b_w/r	N	n	Predicted R_{ac}	Measured R_{ac}	Core Loss	Total Loss
ETD-34 Full Bobbin	1.21 cm^4	3.05	4.37	60	66	0.744 Ω	0.683 Ω	2.35 W	3.89 W
ETD-34 Shape Optimized	1.21 cm^4	3.05	4.37	84	21	0.597 Ω	0.588 Ω	0.96 W	2.28 W
RM-12 Shape Optimized	1.024 cm^4	2.75	2.63	62	20	0.410 Ω	0.446 Ω	0.71 W	1.72 W

core design provides a 46% savings in loss over a design based on a similarly-sized ETD core. Since the designs were optimized individually to give the lowest total loss, the core loss and winding loss are both lower by the same factor of 46% in the RM-core design. The RM core, with the better dimensional ratios, has lower loss despite being smaller (an 18% smaller $W_a A_c$ value). For comparison, a design using the full bobbin of the ETD core was also calculated. This arrangement is calculated to have much higher loss. This is due to the proximity losses caused by the high B field near the gap.

The three designs were fabricated and tested. The optimized winding shapes were roughly approximated with rectangular areas free of windings, implemented by winding tape on that area of the bobbin before starting the litz-wire winding. The inductors were measured with small-signal excitation, using a high-accuracy impedance analyzer as discussed in [13]. The small-signal excitation resulted in core loss much smaller than winding loss, such that the measured ESR was mostly a result of the winding ac resistance. However, core loss at the same voltage drive level was also measured using the same winding on an ungapped core. The measured parallel loss resistance, based on the ungapped core, is included in a model of the final gapped component. Its effect on ESR is subtracted from the measured ESR to give the winding ac resistance, as listed in Table V.

The phase error in the measurement can be expected to give as much as 5% error in ESR [13], given the very high Q (around 500) for these designs. However, for the purposes of comparison between the designs, phase error should not be a problem, since the same phase error would be introduced in each measurement. Additional factors that would be expected to degrade the match between predicted and experimental results include the effect of twist in the litz wire (which increases both dc resistance and eddy-current loss) and the discrepancy between the ideal winding shape and the approximation implemented. Considering these possible error sources, our results match the predicted values very well. The inductor with better dimensional ratios provides a 32% measured improvement in efficiency.

These experiments clearly confirm not only that the optimized designs are superior to the full-bobbin designs, but also that the better window shape of the RM core leads to better performance in a shape-optimized inductor than does the ETD core, despite the smaller core size used.

VI. CONCLUSION

Optimizing the shape of an inductor winding within a fixed core window allows substantial reductions in ac resistance. Also considering the shape of the winding window reveals that choosing shapes with favorable dimensional ratios can lead to additional benefits in reduced losses. The reduction in loss can be used to improve efficiency and reliability, or it may be used to reduce size and cost by moving the design to a smaller core.

For some core shapes, a simple 2-D analysis is adequate. However, for most cores, considering the curvature of the winding is important when considering where to place wire; wire at a larger radius will have a longer turn length and will have higher resistance and eddy current loss, all else being equal. Our new optimization software takes this into account.

Two ratios of core dimensions have been considered: The window aspect ratio b_w/h_w and the ratio of window breadth to centerpost radius b_w/r. Optimal values for a wide range of parameters have been found for both, and compared to typical standard cores. In general, the optimal ratios are lower than those of most standard cores, meaning that choosing cores with relatively small b_w will be advantageous for high-frequency inductors with large ac currents. This has been confirmed by comparing designs on similarly sized RM and ETD cores. The RM-core has more favorable ratios, and the design using it has 32% lower total loss, despite the slightly larger size of the ETD core. Experimental measurements match the predictions well and confirm this shape advantage. Thus, we see that considering dimensional ratios when choosing a core can lead to better performance when a shape-optimized winding is used.

Improved easy-to-use software that is now freely available will make winding-shape optimization available to more users. New features have also improved its accuracy and versatility.

VII. ACKNOWLEDGEMENTS

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